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We present a reflection–positive Euclidean field theory whose analytic continuation yields a Lorentzian quantum field theory on a Friedmann–Robertson–Walker background. A Euclidean cap of finite temporal extent on  $\mathbb{R}_{\tau} \times \mathbb{R}^3$  reproduces the Hartle–Hawking (HH) "no–boundary" state as a special case, with reflection positivity and analytic continuation ensuring identical Lorentzian dynamics. More generally, the framework applies to any reflection–positive Euclidean domain that admits analytic continuation to a unitary Lorentzian theory, providing a uniform reformulation of boundary–based cosmologies. In this view, the Euclidean phase is stationary and pre–geometric, with exponentially suppressed spatial correlations; ordinary space and Lorentzian causality emerge through analytic continuation. The same vacuum state, renormalization structure, and semiclassical dynamics are preserved, so all observable predictions coincide with standard cosmology. Conceptually, the framework illustrates interpretive underdetermination: formally equivalent constructions yield the same empirical content while embodying different ontological commitments. Here, time remains fundamental through the analytic transition, and spatial geometry arises derivatively, clarifying the assumptions underlying cosmological boundary conditions.

**Keywords:** Euclidean quantum field theory; analytic continuation; reflection positivity; interpretive underdetermination; spacetime ontology; foundations of cosmology

### I. INTRODUCTION

Foundational models of the universe's origin often employ a Euclidean path-integral struction—exemplified by  $_{
m the}$ Hartle-Hawking "no-boundary" proposal [9]—as a canonical framework for defining the cosmological wave functional. At the same time, contemporary approaches to quantum gravity increasingly regard spacetime as an emergent construct arising from relational or combinatorial degrees of freedom [1, 3, 5, 10, 11, 14, 15, 17, 18]. Motivated by these developments, we consider a reformulation that is empirically equivalent to the standard no-boundary framework but ontologically distinct: in place of a symmetric spacetime manifold, we take the Euclidean sector to encode a finite-volume temporal domain from which ordinary space emerges upon analytic continuation. Thus, in this reinterpretation, Hawking's question "What is north of the North Pole?" finds its temporal analogue: "Where is the end of east?"

Our aim is not to introduce new geometric structure, but to clarify that what is taken as fundamental—a fully symmetric spacetime or a temporally dominated Euclidean phase—is a matter of interpretive convention rather than empirical necessity. We work within an ordinary reflection—positive Euclidean quantum field theory, defined on a warped background

$$ds_E^2 = d\tau^2 + e^{2\sigma(\tau)}d\mathbf{x}^2,\tag{1}$$

where  $\sigma(\tau) \to -\infty$  as  $\tau \to -\infty$  and  $\sigma'(0) = 0$ . These conditions ensure that the Euclidean action is finite per unit comoving volume: the integral

$$\int_{-\infty}^{0} e^{3\sigma(\tau)} d\tau < \infty, \tag{2}$$

yielding a finite but topologically non-compact "Euclidean cap." Analytically continuing  $\tau \mapsto it$  defines the Lorentzian metric

$$ds_L^2 = -dt^2 + a^2(t) d\mathbf{x}^2, \qquad a(t) = e^{\sigma(it)},$$
 (3)

where  $\sigma$  is assumed analytic in a neighborhood of the real axis and even under reflection. Under these conditions the construction satisfies the Osterwalder–Schrader axioms [12, 13] in the standard sense—reflection positivity and locality—so that the analytically continued theory yields a unitary, causal quantum field theory on the corresponding FRW background. All Lorentzian correlation functions therefore coincide with those of ordinary QFT on the same a(t).

Empirically, this "finite-cap" cosmology is indistinguishable from the Hartle–Hawking framework: the same vacuum state, renormalization structure, and semiclassical dynamics obtain. Conceptually, however, the Euclidean region is reinterpreted. Instead of representing a spacetime "fluctuation into being," it is treated as a stationary, temporally extended pre-geometric phase in which spatial correlations are exponentially suppressed. Ordinary space—and with it, Lorentzian causality—emerges through analytic continuation.

We note that critiques of the Hartle–Hawking proposal address specific contour choices in minisuperspace that can generate runaway modes or non–convergent path integrals [6, 7]. In the present formulation these pathologies do not arise, since the Euclidean segment enters only as a reflection–positive domain whose analytic continuation yields the same Lorentzian wave functional obtained in the standard approach. The finite Euclidean–time (FET) construction therefore retains the stability and boundary regularity of the Hartle–Hawking state while shifting only its ontological interpretation—time is primitive; space is derivative. The following sections develop

the mathematical framework, establish empirical equivalence, and discuss the implications for quantum gravity and the ontology of spacetime.

#### II. MATHEMATICAL FRAMEWORK

Although the Euclidean–Lorentzian correspondence suggests that a temporal compactification should reproduce the same dynamics, this equivalence is not automatic: reflection positivity, boundary smoothness, and analytic continuation of the scale factor must all be checked explicitly. Showing that these conditions hold demonstrates that the Hartle–Hawking framework is analytically complete under reversal of the Euclidean contour.

We formulate the finite Euclidean–time (FET) cosmology entirely within standard Euclidean quantum field theory. The Euclidean manifold of (1) carries a positive–definite Riemannian metric of signature (+,+,+,+). Unlike the original Hartle–Hawking construction, in which the full four–geometry is compact, the present model renders only the Euclidean temporal direction effectively finite through an exponential warp factor. Spatial slices remain topologically  $\mathbb{R}^3$  but contract exponentially as  $\tau \to -\infty$ , where  $e^{\sigma(\tau)} \to 0$ . Consequently, the Euclidean region is finite in proper time yet infinite in spatial extent, and the warp factor dynamically suppresses spatial correlations.

The analytic continuation

$$\tau \mapsto it, \qquad e^{2\sigma(\tau)} \mapsto a^2(t) = e^{2\sigma(it)}, \qquad (4)$$

transforms the metric to Lorentzian signature (-,+,+,+), corresponding to a Friedmann–Robertson–Walker universe with scale factor a(t). The pre–Lorentzian, or finite–Euclidean–time, phase is therefore Riemannian in form but temporally interpreted: a one–dimensional Euclidean domain endowed with exponentially suppressed spatial fibers, not a compact four–manifold.

The technical structure of the framework proceeds as follows: §II A establishes the geometric background, §II B formulates the Euclidean field theory, §II C derives the universal wave functional, and §II D verifies reflection positivity and analytic continuation.

#### A. Geometry of the Euclidean finite-time model

We consider the Euclidean four-manifold

$$M_E \cong \mathbb{R}_{\tau} \times \mathbb{R}^3_{\mathbf{r}},$$
 (5)

equipped with the warped product metric introduced in Eq. (1) of Section I:

$$ds_E^2 = d\tau^2 + e^{2\sigma(\tau)} d\mathbf{x}^2, \tag{6}$$

where the scale function  $\sigma(\tau)$  satisfies

$$\lim_{\tau \to -\infty} \sigma(\tau) = -\infty,$$

$$\sigma'(0) = 0,$$

$$\sigma(\tau) \text{ is nondecreasing for } \tau > 0.$$
(7)

The region  $\tau < 0$  defines a Euclidean "cap" of finite four–volume,

$$Vol(M_E^-) = \int_{-\infty}^0 d\tau \, e^{3\sigma(\tau)} < \infty, \tag{8}$$

ensuring that all Euclidean functional integrals are well defined. The slice  $\tau=0$  serves as the equator, characterized by vanishing extrinsic curvature  $K|_{\tau=0}=3\sigma'(0)=0$ . For  $\tau>0$ , the warp factor  $e^{\sigma(\tau)}$  increases smoothly, so that spatial separations acquire finite extent and ordinary geometrical structure emerges continuously.

#### B. Euclidean field theory

For definiteness, consider a real scalar field  $\phi$  with local polynomial potential  $V(\phi)$  on  $(M_E, g_E)$ , governed by the Euclidean action

$$S_E[\phi;\sigma] = \int d\tau \, d^3x \, e^{3\sigma(\tau)}$$

$$\times \left[ \frac{1}{2} (\partial_\tau \phi)^2 + \frac{1}{2} e^{-2\sigma(\tau)} |\nabla \phi|^2 + V(\phi) \right].$$
(9)

The generating functional for Euclidean correlation functions is

$$Z[J] = \int \mathcal{D}\phi \, \exp\left(-S_E[\phi; \sigma] + \int d\tau \, d^3x \, J(\tau, \mathbf{x}) \, \phi(\tau, \mathbf{x})\right). \tag{10}$$

Locality and polynomial boundedness of  $V(\phi)$  ensure that Z[J] defines a reflection–positive Schwinger functional in the sense of Osterwalder and Schrader [12, 13]. In the cap region ( $\tau < 0$ ), the exponential suppression  $e^{\sigma(\tau)} \to 0$  effectively eliminates spatial gradients, so that Eq. (9) factorizes into independent one–dimensional path integrals for each comoving spatial label  $\mathbf{x}$ . Spatial correlators therefore decay exponentially for  $\tau < 0$ , realizing the "no–space" pre–geometric regime introduced in Section I.

#### C. Universal wave functional at the equator

The wave functional on the equatorial slice  $\tau = 0$  is obtained from the Euclidean path integral with boundary value  $\phi(0, \mathbf{x}) = \varphi(\mathbf{x})$ :

$$\Psi[\varphi] = \int_{\phi|_{\tau=0}=\varphi} \mathcal{D}\phi \ e^{-S_E[\phi;\sigma]}. \tag{11}$$

For  $\tau<0$ , the suppression of spatial gradients implies that  $\Psi[\varphi]$  factorizes over  ${\bf x}$  into quasi–Gaussian ground

states of the one–dimensional dynamics along  $\tau$ . Formally,

$$\Psi[\varphi] \propto \exp\left(-\frac{1}{2}\int \frac{d^3k}{(2\pi)^3} \,\omega_k^{\text{eff}} \,|\tilde{\varphi}_k|^2\right),$$
 (12)

where  $\omega_k^{\text{eff}}$  is determined by the cap propagator associated with (9). This  $\Psi$  serves as the universal "vacuum" state for the subsequent Lorentzian evolution.

#### D. Reflection positivity and analytic continuation

Define the reflection operator  $\Theta: \tau \mapsto -\tau$ ,  $(\Theta\phi)(\tau, \mathbf{x}) = \phi(-\tau, \mathbf{x})$ . Because  $\sigma(\tau)$  is even and  $\sigma'(0) = 0$ , the Euclidean action (9) satisfies  $S_E[\Theta\phi; \sigma] = S_E[\phi; \sigma]$ . For any functional  $F[\phi]$  supported in  $\tau > 0$ , the Osterwalder–Schrader inner product

$$\langle F, \Theta F \rangle_{\text{OS}} = \int \mathcal{D}\phi \, \overline{F[\Theta\phi]} F[\phi] \, e^{-S_E[\phi;\sigma]} \ge 0$$
 (13)

holds under the standard assumptions of reflection positivity and integrability. The OS reconstruction theorem then provides a Hilbert space, vacuum vector, and field operators satisfying the Wightman axioms on the Lorentzian continuation,

$$ds_L^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \qquad a(t) = e^{\sigma(it)},$$
 (14)

where  $\sigma$  is assumed analytic in a neighborhood of the real axis. Time–ordered correlation functions obtained from (14) coincide with those of standard quantum field theory on the same FRW background, confirming empirical equivalence.

The mathematical results of this section demonstrate that the finite Euclidean–time cosmology constitutes a consistent Euclidean field theory obeying the same axioms as ordinary QFT, while introducing a distinct temporal ontology that will be examined further in Sections III and IV.

# III. EMPIRICAL EQUIVALENCE AND INDISTINGUISHABILITY

Having established the internal consistency of the Euclidean construction in Section II, we now show that all Lorentzian observables derived from the finite Euclidean–time (FET) model coincide with those of conventional quantum field theory on a Friedmann–Robertson–Walker (FRW) background. This section analyses the free theory (§III A), interacting renormalized theory (§III B), and semiclassical gravitational coupling (§III C).

### A. Free-field correspondence

For  $V(\phi) = \frac{1}{2}m^2\phi^2$ , the Fourier transform  $\phi(\tau, \mathbf{x}) = \int d^3k (2\pi)^{-3}\phi_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}$  reduces the Euclidean action (9)

to independent modes,

$$S_E[\phi_k] = \frac{1}{2} \int d\tau \, e^{3\sigma(\tau)} \Big[ (\partial_\tau \phi_k) (\partial_\tau \phi_{-k}) + \left( m^2 + e^{-2\sigma(\tau)} k^2 \right) \phi_k \phi_{-k} \Big]. \tag{15}$$

Each mode  $\phi_k(\tau)$  satisfies the one-dimensional Euclidean equation of motion,

$$\partial_{\tau}^{2}\phi_{k} + 3\sigma'(\tau)\partial_{\tau}\phi_{k} - \left[m^{2} + e^{-2\sigma(\tau)}k^{2}\right]\phi_{k} = 0. \quad (16)$$

The regular solution that decays for  $\tau \to -\infty$  determines the Gaussian width  $\omega_k^{\text{eff}}$  in Eq. (12). Under the analytic continuation  $\tau \mapsto it$ , the equation becomes

$$\ddot{\phi}_k + 3\frac{\dot{a}}{a}\dot{\phi}_k + \left(\frac{k^2}{a^2} + m^2\right)\phi_k = 0,$$
 (17)

which is precisely the mode equation for a standard scalar field on the FRW metric (14). By choosing the Euclidean cap profile  $\sigma(\tau)$  such that  $\omega_k^{\text{eff}} = \omega_k^{\text{BD}}$  (the Bunch–Davies frequency), the equatorial wave functional (12) reproduces the conventional vacuum two–point function,

$$\langle 0|\phi(t,\mathbf{x})\phi(t',\mathbf{y})|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{2\omega_k^{\mathrm{BD}}} e^{-i\omega_k^{\mathrm{BD}}|t-t'|}.$$
(18)

All free–field observables and power spectra therefore coincide with those of standard cosmology under the same analytic continuation.

#### B. Interacting and renormalized theory

Local interactions and renormalization proceed identically because the Osterwalder–Schrader axioms remain valid. Counterterms in perturbation theory are integrals of local curvature and field polynomials, independent of global geometric properties. Explicitly,

$$\delta S_E = \int \sqrt{g_E} \left( \delta \Lambda + \delta \xi R_E \phi^2 + \delta m^2 \phi^2 + \delta \lambda \phi^4 + \cdots \right), (19)$$

where  $R_E$  is the scalar curvature of  $g_E$  in Eq. (6). Since the curvature invariants are bounded [cf. Eq. (7)], the divergences and renormalization group flow coincide with those of ordinary Euclidean field theory on a smooth background. Upon analytic continuation, the same counterterms yield the Lorentzian effective action and stress tensor. Thus, the finite Euclidean—time geometry neither modifies nor obstructs the standard renormalized QFT structure.

#### C. Semiclassical gravity and bounded curvature

Coupling the renormalized stress tensor to gravity gives the semiclassical Einstein equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle_{\Psi}, \tag{20}$$

where the expectation value is taken in the state  $\Psi[\varphi]$  defined by Eq. (11). In the Lorentzian regime,  $\langle T_{\mu\nu}\rangle_{\Psi}$  matches the standard vacuum expectation value in the background (14). In the Euclidean cap,  $\sqrt{g_E} \sim e^{3\sigma(\tau)}$  ensures that all curvature integrals remain finite even as  $\sigma(\tau) \to -\infty$ :

$$|R_E| < C_1 + C_2 e^{-2\sigma_{\min}}, \qquad \int_{-\infty}^0 |R_E| \sqrt{g_E} \, d\tau d^3x < \infty.$$
 (21)

Hence the finite Euclidean–time geometry is geodesically complete and nonsingular, with the usual FRW dynamics recovered for t>0.

#### D. Summary of empirical equivalence

To summarize, Eqs. (18)–(21) show that:

- (i) The reconstructed Lorentzian theory satisfies the same field equations and two-point functions as standard QFT.
- (ii) Interacting and renormalized dynamics are unaffected by the finite Euclidean–time cap.
- (iii) Gravitational backreaction and curvature remain finite and reproduce the standard semiclassical limit.

All measurable predictions of the finite Euclidean–time universe therefore coincide with those of conventional cosmology. Any distinction between the two resides solely in their ontological interpretation, a theme explored in Section IV.

#### IV. QUANTUM GRAVITY COMPATIBILITY

Section III established that the finite Euclidean–time (FET) model reproduces all empirical predictions of standard quantum field theory and semiclassical gravity. We now show that the same structure naturally accommodates a quantum–gravitational interpretation. Three complementary formulations are considered: the Euclidean gravitational path integral (§IV A), the canonical Wheeler–DeWitt framework (§IV B), and discrete or renormalization–group approaches (§IV C).

## A. Euclidean gravitational path integral

Promoting the warp function  $\sigma(\tau)$  of Eq. (6) to a dynamical metric variable yields the gravitational Euclidean action

$$S_E[g_E, \phi] = -\frac{1}{16\pi G} \int R_E \sqrt{g_E} \, d^4x + S_E[\phi; \sigma],$$
 (22)

where  $S_E[\phi; \sigma]$  is the matter action from Eq. (9). The full partition function integrates over smooth, regular Euclidean geometries,

$$Z = \int_{\text{regular}} \mathcal{D}g_E \,\mathcal{D}\phi \,e^{-S_E[g_E,\phi]}.$$
 (23)

Boundary conditions are specified by the induced three–metric  $h_{ij}$  and field configuration  $\varphi$  on the equator  $\tau = 0$ . The corresponding wave functional,

$$\Psi[h_{ij}, \varphi] = \int_{\substack{g_E|_{\tau=0}=h\\\phi|_{\tau=0}=i\varphi}} g_E \mathcal{D}\phi \ e^{-S_E[g_E, \phi]}, \qquad (24)$$

is mathematically identical to the Hartle–Hawking no–boundary state [8, 9], except that the Euclidean manifold now possesses a finite Euclidean temporal domain rather than a compact four–geometry [cf. Eq. (8)]. The suppression  $e^{\sigma(\tau)} \rightarrow 0$  regularizes both infrared and ultraviolet regions of the gravitational path integral, providing a natural geometric cutoff without additional fields or parameters.

The saddle point of (23) satisfies the Euclidean Einstein equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(E)}, \tag{25}$$

whose regular solution corresponds to the finite Euclidean–time geometry of Eqs. (6)–(7). Analytic continuation of this saddle reproduces the Lorentzian FRW universe of Eq. (14).

#### B. Canonical Wheeler-DeWitt picture

In the minisuperspace reduction, the metric  $ds_E^2 = d\tau^2 + a^2(\tau)d\mathbf{x}^2$  with  $a(\tau) = e^{\sigma(\tau)}$  yields the one-dimensional gravitational action

$$S_E[a] = \frac{3\pi}{4G} \int d\tau \left[ a(a')^2 - a + \frac{\Lambda}{3}a^3 \right].$$
 (26)

Quantization leads to the Wheeler–DeWitt equation,

$$\left[ -\frac{\hbar^2}{2M_P^2} \frac{d^2}{da^2} + U(a) \right] \Psi(a) = 0, \tag{27}$$

where  $M_P = (3\pi/2G)^{1/2}$  and

$$U(a) = \frac{3\pi}{4G} \left( a^2 - \frac{\Lambda}{3} a^4 - \frac{8\pi G}{3} a^4 \rho(a) \right)$$
 (28)

is the effective potential including the matter energy density  $\rho(a)$ . The Euclidean region of the finite Euclidean–time sector corresponds to U(a)>0 (classically forbidden), producing the exponentially suppressed wave functional

$$\Psi(a) \propto \exp\left[-\int^a \sqrt{2M_P^2 U(a')} \, da'\right], \qquad (29)$$

while the Lorentzian region U(a) < 0 yields oscillatory solutions. Equation (29) is precisely the WKB limit of the Euclidean cap integral (24), providing a canonical realization of the finite Euclidean–time geometry within quantum cosmology.

# C. Discrete and renormalization—group perspectives

The finite Euclidean–time structure is compatible with discrete quantum gravity approaches. In spin–foam or loop–quantum–gravity formulations, the equatorial three–surface corresponds to a boundary spin network, and the Euclidean cap in Eq. (23) is represented by a finite four–complex (foam) whose amplitude A[boundary] generates the same wave functional as Eq. (24). Reflection positivity of the continuum theory [Eq. (13)] ensures a real, positive amplitude in the discrete setting.

In the asymptotic–safety program, the Euclidean cap provides a geometric regularization for the gravitational functional renormalization group. Because the Euclidean temporal direction is finite in measure, the heat–kernel trace used to define the running Newton and cosmological constants remains finite, realizing the ultraviolet fixed point in a natural way.

#### D. Summary of quantum-gravitational features

The finite Euclidean–time model therefore:

- (i) reproduces the Hartle-Hawking no-boundary wave functional with a finite Euclidean temporal domain in place of a compact four-geometry,
- (ii) satisfies the Wheeler–DeWitt equation (27) with a smooth transition between Euclidean and Lorentzian regimes, and
- (iii) provides a finite, reflection—positive Euclidean measure suited to spin—foam and asymptotic—safety formulations.

No additional fields, parameters, or symmetries are required. The framework is thus fully compatible with existing quantum–gravity programs while retaining the parsimony and empirical equivalence established in Section III.

# V. DISCUSSION AND PHILOSOPHICAL IMPLICATIONS

The preceding sections have shown that the finite Euclidean–time (FET) universe—defined by the Euclidean geometry of Eq. (6) and the reflection–positive field theory of Eq. (9)—reproduces all empirical consequences of conventional quantum field theory and semiclassical

gravity (Sections III–IV). The distinction lies entirely in its underlying ontology. The transformation to Euclidean time may appear as a mere sign or contour flip, but its internal consistency is non-trivial: reflection positivity, smooth boundary matching, and analytic continuation of the geometry all survive intact. This confirmation establishes that the no-boundary framework remains self-consistent under temporal Euclideanization, revealing a previously unarticulated symmetry between its spatial and temporal formulations. Here we summarize this reinterpretation and its broader conceptual significance.

## A. Ontological reinterpretation

In the conventional Hartle–Hawking picture [8, 9], spacetime is a compact four–manifold whose Euclidean and Lorentzian domains are joined by analytic continuation, treating space and time symmetrically at the origin. By contrast, the finite Euclidean–time model posits that only the Euclidean temporal dimension is fundamental and finite in extent, while spatial relations are emergent properties that become meaningful when the warp factor  $e^{\sigma(\tau)}$  attains finite value. The pre–geometric Euclidean cap is therefore not "nothing" but an eternal potential—a stationary configuration lacking spatial separation yet supporting temporal order. The Lorentzian universe arises smoothly when this potential admits commutative spatial directions, rendering geometry and causality emergent rather than primitive.

This reformulation replaces the notion of "creation from nothing" with a regular, boundaryless Euclidean phase of finite measure. Because reflection—positive Euclidean theories admit analytic continuation to unitary Lorentzian ones, the standard physics of fields, vacuum fluctuations, and gravity follow automatically from the formal machinery of Euclidean QFT.

#### B. Relation to time asymmetry and emergence

The asymmetry between a finite Euclidean temporal domain and emergent spatial relations provides a natural foundation for temporal orientation. The Euclidean cap defines a unique direction of analytic continuation, selecting an arrow of time without inserting explicit time–reversal violations. Spatial locality—and consequently causal structure—appear only in the Lorentzian regime. This viewpoint harmonizes with relational and process ontologies of spacetime [2, 16] while remaining consistent with the operational content of relativistic physics. In this sense, the FET framework supplies a mathematical realization of the idea that temporal order may be more fundamental than spatial geometry.

# C. Empirical indistinguishability and interpretive plurality

As demonstrated in Section III, all measurable correlators, vacuum effects, and gravitational observables coincide with those predicted by standard cosmology. No experiment could distinguish the finite Euclidean—time model from the conventional no—boundary universe. The difference is therefore interpretive rather than predictive—analogous to the relation between Schrödinger and Heisenberg pictures in quantum mechanics. The two descriptions share identical empirical structure while attributing different ontological roles to time and space. The model thus exposes a layer of conceptual underdetermination that is often implicit in cosmological reasoning.

#### D. Philosophical significance and outlook

The finite Euclidean—time (FET) cosmology provides a concrete illustration of how a shift in ontological priority—placing time before space—can be realized within established physics without altering empirical content. "Nothing" in cosmology thus need not denote the absence of being, but the absence of spatial relations within a temporally ordered background. The FET framework links technical quantum—cosmological formalisms with metaphysical debates on the fundamentality of time, the nature of beginnings, and the emergence of spacetime

from non–geometric processes. It also demonstrates that interpretive plurality—distinct ontologies within a shared mathematical structure—need not threaten scientific realism but can clarify which aspects of a theory describe the world and which reflect representational choices.

This situation exemplifies the broader theme of *interpretive underdetermination* in the philosophy of cosmology [4, 19]: formally equivalent frameworks can embody different ontological assumptions while remaining empirically indistinguishable. The FET model preserves all observable structure of the Hartle–Hawking no–boundary proposal (and any reflection–positive Euclidean domain analytically continued to Lorentzian spacetime [6, 7]) yet relocates explanatory priority from spacetime symmetry to temporal fundamentality. Recognizing this underdetermination reframes the "origin of the universe" problem as primarily interpretive rather than dynamical.

Future work may extend this formulation in several directions: incorporation of spinor and gauge fields within the reflection–positive scheme, explicit construction of Euclidean gravitational instantons realizing Eq. (23), and investigation of entanglement and information flow across the Euclidean–Lorentzian junction. More broadly, the framework invites exploration of temporal finiteness within quantum information, relational time, and process–based approaches to quantum gravity—suggesting that questions about cosmic beginnings may reveal less about new physics than about the conceptual lenses through which existing formalisms are interpreted.

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